

Effects of adverse pressure gradient on Reynolds stresses inturbulent boundary layers

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Abstract: The effects of adverse pressure gradient (APG) on Reynolds stresses in turbulent boundary layers (TBLs) with APG were analyzed. The difficulty of this work was attributable to the Reynolds stresses in TBLs with APG under two combined effects, i. e. : effect of upstream flow and effect of APG. The effect of upstream flow is an inherent effect no matter pressure gradient exists or not. The individual effect was analyzed from absolute developments of Reynolds stresses in TBLs with zero pressure gradient (ZPG) firstly. Effect of APG was then analyzed from absolute developments of Reynolds stresses in TBLs with APG. Result showed that, for absolute development of mean streamwise Reynolds stresses, APG accelerated its development in TBL with ZPG; for absolute development of mean normal or shear Reynolds stresses, APG increased their magnitude in the outer part, and decreased their extent of large value region.

Key words: turbulent boundary layer; adverse pressure gradient; zero pressure gradient; Reynolds stresses; flat plate

CLC number: V231.3; O357.5+2

Document code: A

Nomenclature

Letters

Re_θ	Reynolds number based on momentum thickness
U	streamwise mean velocity, m/s
V	normal mean velocity, m/s
U_{ref}	reference inlet mean velocity, m/s
u'	streamwise fluctuation velocity, m/s
v'	normal fluctuation velocity, m/s
u_τ	friction velocity, m/s
$u_{\tau 0}$	reference friction velocity, m/s

Subscripts

a	absolute development
r	relative development

Acronyms

APG	adverse pressure gradient
DNS	direct numerical simulation
HWA	hot-wire anemometry
TBL	turbulent boundary layer
ZPG	zero pressure gradient

Turbulent boundary layers (TBLs) with adverse pressure gradient (APG) exist in lots of important industrial devices such as airfoils and diffusers, and relates to their efficiency as well as stability. Therefore, many basic investigations are dedicated to investigating the behaviors

of TBLs with APG^[1-12]. Research methods used in these investigations are mainly experiments or numerical simulations. This is because it's very difficult to get general interpretations from the transport equations for Reynolds stresses, due to mean streamwise pressure gradient does not

Received: 2015-05-18

Foundation item: Natural Science Foundation of Shanghai (15ZR1422500);

Initiative Projects of Shanghai Jiao Tong University Young Teachers (AF4130024)

Citation: LUO Xiaotian, ZHANG Ce, MA Wei, et al. Effects of adverse pressure gradient on Reynolds stresses inturbulent boundary layers[J]. Journal of Aerospace Power, 2017, 32(2): 463-469.

directly appear in those transport equations.

About effect of APG on Reynolds stresses in TBLs, researches mainly focused on the effect of APG on the maximum value in the profiles of streamwise Reynolds stresses $u'u'$. In both numerical and experimental researches, an outer peak usually appears in the profile of $u'u'$ in a TBL with APG^[5, 13]. Shah, et al.^[14] considered that this outer peak is triggered by the APG by summarizing absolute development of $u'u'$ in TBLs with APG. After considering that outer peak even exists in the profile of $u'u'$ in a TBL with zero pressure gradient (ZPG) if this TBL continually develops in streamwise direction, now it's widely accepted that APG accelerates the appearance of the outer peak, instead of being a trigger^[15].

However effects of APG on Reynolds stresses are still not clearly understood, especially effects of APG on absolute developments of Reynolds stresses both in inner part and in outer part. Therefore it needs to be investigated systematically. Compared with relative developments of Reynolds stresses that non-dimensionalized by local friction velocity, absolute developments of Reynolds stresses could avoid the influences of uncertainty in friction velocity, whose accuracy is limited to no better than 2% in experiments usually measured by hot-film anemometry.

The purpose of this research is investigating effects of APG on Reynolds stresses in TBLs using numerical and experimental results in the literature. To remove influences of other factors (e. g. curvature), the results on flat plates are focused.

The difficulty of investigating effects of APG on Reynolds stresses in TBLs comes from the fact that Reynolds stresses in TBLs with APG on flat plates are under two combined effects (Fig. 1). Effect I is the effect of upstream flow. It reflects the history influence of upstream flow. This is an inherent effect no matter pressure gradient exists or not. Effect II is the effect of adverse pressure gradient. This is a local effect. Therefore for investigating indi-

vidual effect of APG, firstly it is necessary to investigate the individual effect of upstream flow.

In this paper, relative developments of Reynolds stresses in TBLs with ZPG are presented firstly, which are commonly presented in previous investigations. Subsequently, absolute developments of Reynolds stresses in TBLs with ZPG are investigated, from which individual effect of upstream flow could be seen clearly. Finally, absolute developments of Reynolds stresses in TBLs with APG are investigated, from which effect of APG could be analyzed.

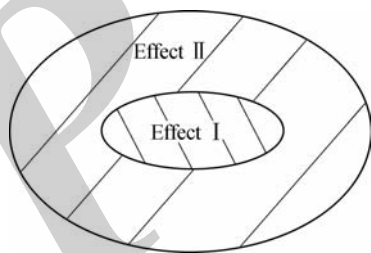


Fig. 1 Two combined effects in TBLs with APG

1 Relative developments of Reynolds stresses in TBLs with ZPG

In this section, relative developments of Reynolds stresses in TBLs with ZPG which are commonly presented in previous investigations, are presented and analyzed. In order to eliminate influences of uncertainty in experiments and turbulent model in numerical simulations, here we seek results of direct numerical simulations (DNS) in the first place. One of DNS results, which is available for anyone and could be obtained in authors' personal website, is Schlatter and Örlü^[16]. The Reynolds number based on momentum thickness $Re_\theta = 670 \sim 4060$ in streamwise direction. Fig. 2 shows the sketch of their turbulent boundary layer (TBL).

Figure 3 shows the developments of Reynolds stresses non-dimensionalized by local friction velocity, e. g. relative developments of Reynolds stresses in TBLs with ZPG. For $(u'u')_r = \sqrt{u'^2}/u_\tau$ (Fig. 3(a)), one obvious phenomenon is that an inner peak appears very near the wall. This inner peak locates at $y^+ \approx 15$; and

this location is almost independent with Reynolds number. The magnitude value at this inner peak increases with Reynolds number.

For $(v'v')_r = \sqrt{v'^2}/u_\tau$ (Fig. 3(b)), the profile initially rises until its maximum value and then decreases slowly. The maximum value and its location generally increase with Reynolds number increasing. For $(u'v')_r = -\sqrt{u'v'}/u_\tau$ (Fig. 3(c)), its trend is similar to that of $(v'v')_r$, shown in Fig. 3(b).

DNS results are limited, due to the limitation of the capability of current computing. Therefore trends of $(u'u')_r$, $(v'v')_r$ and $(u'v')_r$ with Reynolds number continually increasing cannot be shown in Fig. 3. Fortunately, these trends can be discovered from the experimental results (exp.) summarized by Fernholz and Finley^[17] as shown in Fig. 4. For $(u'u')_r$ (Fig. 4(a)) when Reynolds number increases, an outer peak

gradually appears in the outer layer, and the value of $(u'u')_r$ at this outer peak gradually increases. This phenomenon has also been mentioned by the recent review of Marusic, et al.^[6] It needs to note that the value of $(u'u')_r$ at the outer peak is still smaller that at the inner peak. The difference between the values at the inner and outer peaks decreases with Reynolds number. For $(v'v')_r$ and $(u'v')_r$ Fig. 4(b) – Fig. 4(c), the trends are similar to those in the corresponding Fig. 3(b) – Fig. 3(c).

To show together the trend when Reynolds number increases discussed above, the “profile 1” in each sub-figure of Fig. 3 represents the trend reported in Fernholz and Finley^[17].

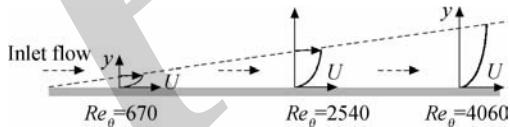


Fig. 2 Sketch of the TBL of Schlatter & Örlü^[16]

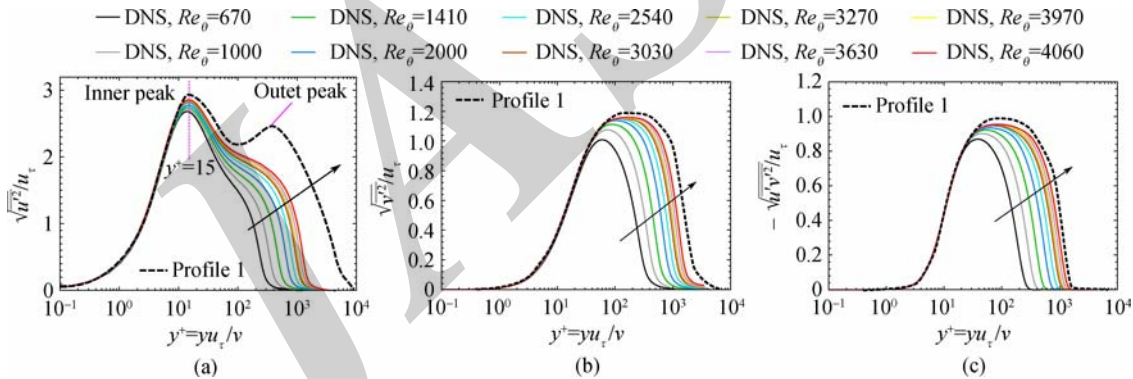


Fig. 3 Relative developments of Reynolds stresses with Re_θ in TBLs with ZPG (DNS results are from Schlatter and Örlü^[16]; arrows denote Reynolds number increasing directions)

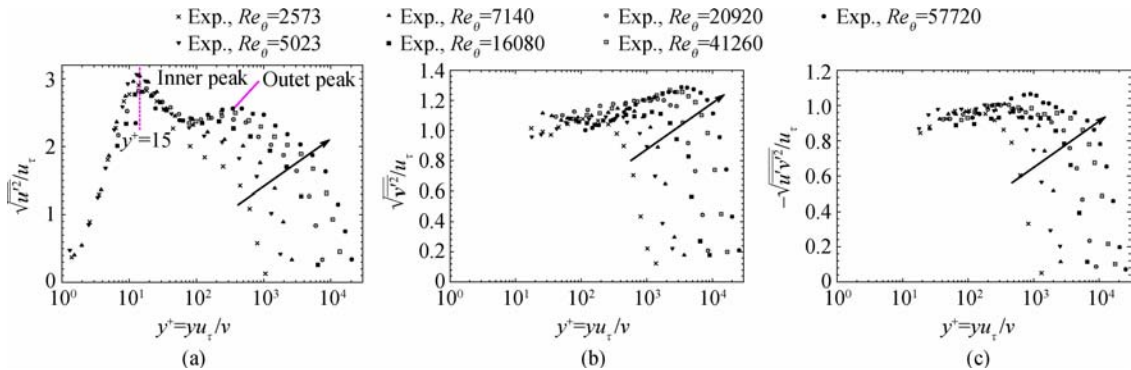


Fig. 4 Relative developments of Reynolds stresses with Re_θ in TBLs with ZPG (Experiments results are from Fernholz and Finley^[17]; arrows denote Reynolds number increasing directions)

2 Absolute developments of Reynolds stresses in TBLs with ZPG

In this section, absolute developments of Reynolds stresses in TBLs with ZPG are presented, from which individual effects of upstream flow could be seen clearly.

Figure 5 shows the absolute developing of Reynolds stresses shown in Fig. 3, including the DNS results of Schlatter and Örlü^[16], as well as “profile 1” that represents the trend when TBL continues developing. In this figure, the reference friction velocity $u_{\tau 0}$ is the value at the first station where $Re_{\theta} = 670$.

For $(u'u')_a = \sqrt{u'^2}/u_{\tau 0}$ in Fig. 5 (a), the boundary layer can be divided into inner part and outer part. The inner part is the region where an inner peak exists. With the streamwise development (i. e. with Reynolds number increasing),

$(u'u')_a$ at the inner peak decreases in contrast to the development of $(u'u')_r$ shown in Fig. 3(a); and the location of this inner peak increases slightly. In the outer part, $(u'u')_a$ increases and the extent of outer part expands gradually with the streamwise development. The outer peak in the profile of streamwise Reynolds stress already exists without APG. Therefore the APG is not a trigger for the appearance of this outer peak.

For $(v'v')_a = \sqrt{v'^2}/u_{\tau 0}$ in Fig. 5 (b), its maximum value decreases with the streamwise development, and its location increases slightly. At the same time, extent of the high value region increases considerably.

For $(u'v')_a = -\sqrt{u'v'}/u_{\tau 0}$ in Fig. 5 (c), its trend is similar to that of $(v'v')_a$ in Fig. 5(b) and discussed above.

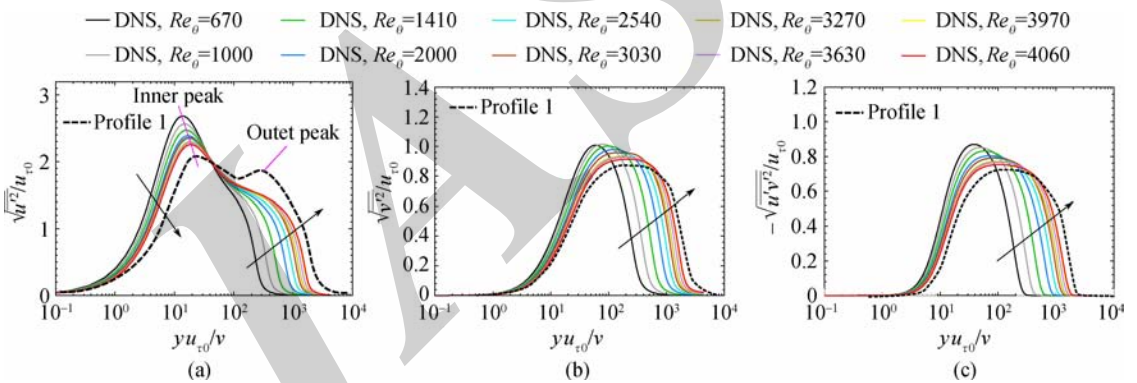


Fig. 5 Absolute developments of Reynolds stresses in a TBL with ZPG (“profile 1”. $u_{\tau 0}$ denotes friction velocity at the first station; arrows denote streamwise directions)

3 Absolute developments of Reynolds stresses in TBLs with APG

After analyzing effect of upstream flow in section 3, in this section we analyze the effect of APG by re-examining the previous results in TBLs with APG on flat plates in the literature, in which the APG is generated by curvature of an upper wall. For $(u'u')_a$, we examine DNS results of Spalart and Watmuff^[1] are examined, as well as experimental results of Materny, et al.^[13] and Bernard, et al.^[18] measured by hot-wire anemometry (HWA), as shown in Fig. 6.

In this figure velocity scale $u_{\tau 0}$ is the value at reference station in each case, and thus here $(u'u')_a = \sqrt{u'^2}/u_{\tau 0}$. For $(v'v')_a$ and $(u'v')_a$, experimental results of Elsberry, et al.^[5] measured by HWA, as shown in Fig. 7. In this figure, the velocity scale U_{ref} is the inlet reference velocity, and thus $(v'v')_a = \overline{v'^2}/U_{\text{ref}}^2$ and $(u'v')_a = -\overline{u'v'}/U_{\text{ref}}^2$. $(x - x_0)/\theta_0$ is the dimensionless number of the direction of streaming. Many distances in the direction of streaming are referred to the virtual origin of the flow (x_0), and θ_0 is the local momentum thickness.

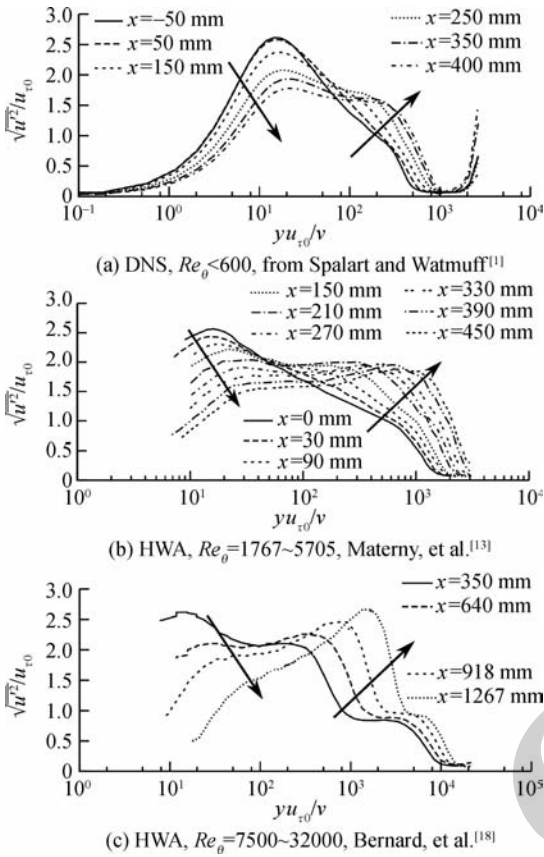


Fig. 6 Absolute developments of streamwise Reynolds stress in TBLs with APG ($u_{\tau 0}$ denotes friction velocity at first station; arrows denote streamwise directions)

Figure 6(a) shows clearly the effect of the upstream flow as shown in Fig. 5(a). In this case, the outer peak does not yet appear even at the last measurement station.

Figure 6(b) also shows clearly the effect of upstream flow as shown in Fig. 5(a), especially in the former stations. An outer peak appears after the station $x=210$ mm. At this station, $(u'u')_a$ at outer peak is still smaller than that at the inner peak, as the “profile 1” in Fig. 5(a). And then $(u'u')_a$ at the outer peak augments gradually when TBL develops in streamwise direction. At the station $x=270$ mm, $(u'u')_a$ at outer peak is nearly equal to that at the inner peak. At the last three stations $x=330, 390$ mm and 450 mm, $(u'u')_a$ at outer peak is already larger than that at the inner peak. In order to represent this trend, “profile 2” in Fig. 5(a) and replot in Fig. 8(a) are added. The Reynolds number Re_θ in Materny, et al.^[13] (Fig. 6(b)) is from 1767 to 5705. The maximum value 5705 is nearly an or-

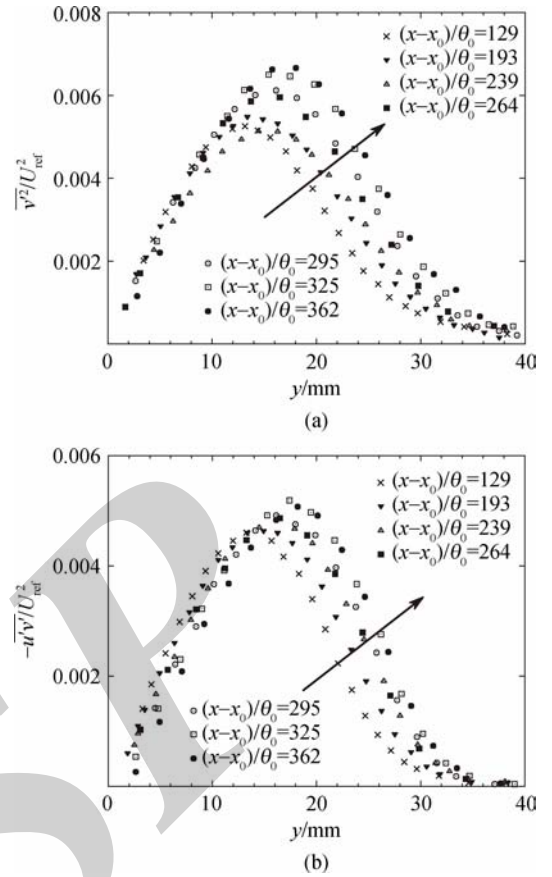


Fig. 7 Absolute developments of Reynolds stresses in a TBL with APG (HWA, $Re_\theta = 6280 \sim 13933$ are from Elsberry, et al.^[5]; U_{ref} denotes inlet mean velocity; arrows denote streamwise directions)

der of magnitude smaller than the maximum value in Fig. 4(a) 57 720. Therefore we infer that the APG accelerates the appearance of the outer peak in the profile of $(u'u')_a$.

Figure 6(c) also shows clearly the effect of upstream flow shown in Fig. 5(a). At first station $x=350$ mm, outer peak does not yet appear. At other stations $x=640, 918, 1267$ mm, outer peak already appears. When TBL develops, $(u'u')_a$ at inner peak continued decreases, while at outer peak continued increases. At station $x=640$ mm, $(u'u')_a$ at outer peak is nearly the same as that at the inner peak, as “profile 1” in Fig. 8(a). At station $x=918$ mm, $(u'u')_a$ at outer peak is already larger than that at the inner peak, as “profile 2” in Fig. 8(a). At station $x=1267$ mm, inner peak disappears. In order to present this trend, we add “profile 3” in Fig. 8(a).

In Fig. 7, maximum values of $(v'v')_a$ and $(u'v')_a$ increase, and the extents of high value region narrow when the TBL continues developing.

This trend can be represented by “profile 2” and “profile 3” in the corresponding Fig. 8(b)—Fig. 8(c).

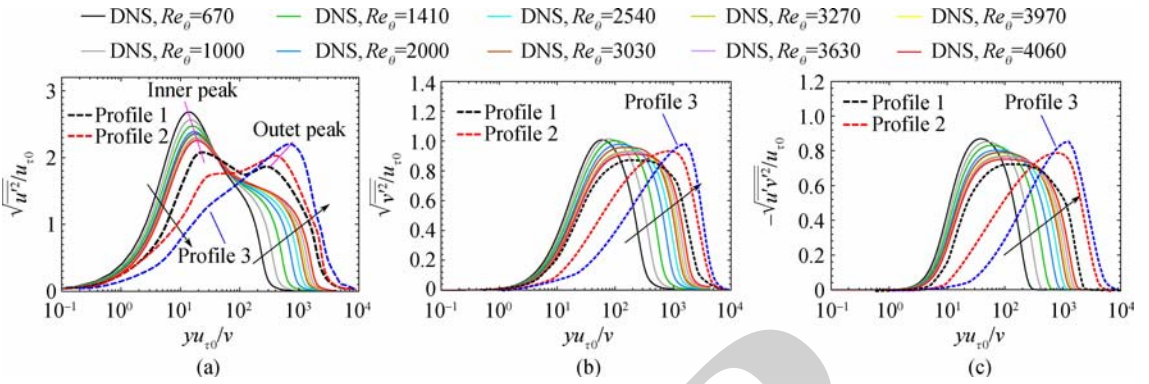


Fig. 8 Absolute developments of Reynolds stresses in a TBL with ZPG. (add “profile 2” and “profile 3” based on Fig. 5. $u_{\tau 0}$ denotes friction velocity at the first station; arrows denote streamwise directions)

4 Conclusions

In this paper, the individual effect of upstream flow on Reynolds stresses in TBLs is firstly analyzed through absolute developments of Reynolds stresses in TBLs with ZPG. Then effect of APG on Reynolds stresses in TBLs is analyzed from absolute developments of Reynolds stresses in TBLs with APG.

1) For $(u'u')_a$, APG accelerates the development in a TBL with ZPG. In the inner part, APG accelerates the decrease of $(u'u')_a$. In the outer part, APG accelerates the increase of $(u'u')_a$. Additionally, if APG is strong enough, out peak occurs early comparing with that in TBLs with ZPG. At outer peak with magnitude of the APG increasing, magnitude of $(u'u')_a$ increases, and its location departs further from the wall.

2) For $(v'v')_a$, APG accelerates the decrease of $(v'v')_a$ in the inner part of boundary layer. In the outer part, APG decreases the extent of large value region, and increases the maximum value. If APG is strong enough, a peak occurs in the outer part. The magnitude of APG is larger, the magnitude of $(v'v')_a$ at outer peak is larger, and the location of outer peak departs further from the wall.

3) For $(u'v')_a$, its trend under APG is simi-

lar to that of $(v'v')_a$ discussed above.

These conclusions are the foundation to investigate TBLs with APG in turbomachinery that relates to efficiency and stability, such as the TBL along blade suction side at mid-span of a linear compressor cascade.

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(编辑:王碧珺)